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Article

Changes in the Ghanaian schools' mathematics curriculum – Exploring the need to raise awareness in the integration of the history of mathematical concepts as a pedagogical tool

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Abstract: In light of the frequent changes in the Ghanaian mathematics curriculum, our study aims to explore the teachers' awareness of integrating the history of mathematical concepts as a pedagogical strategy in teaching. To achieve this, we employed the interpretive paradigm and used the qualitative inductive approach of an explorative case study with eight participating secondary school mathematics teachers. The interviews were conducted and transcribed for analysis using content analysis. The results indicated that while most teachers were enthusiastic about using the history of mathematical concepts as a strategy, they hardly employed it as a critical approach to teaching mathematics. Those who were aware of it merely used it as a form of introduction to their lessons—this limited learners' engagement level in the history of mathematical concepts in learning mathematics. Conclusions and recommendations were discussed.

Keywords: Awareness, Curriculum, History of mathematical concepts, Pedagogical tool, Strategy

1. Introduction

There is an urgent and continuous need for educational reforms in Ghana, driven by accelerated development. This has led to frequent changes in the educational system from the pre-colonial era to the present day. The country has experienced different eras of reforms since independence, including the accelerated development of 1951 and the Educational Act of 1961, the new 1974 education reform, followed by the 1987 education reform and the 2007 education reforms (Adu-Gyamfi, 2016). Following these is the 2020 education reform for basic schools (NaCCA, 2020). As the general educational system transforms, so does the mathematics curriculum, highlighting the need for continuous adaptation and improvement.

The mathematics curriculum underwent a significant transformation in content but not much in pedagogy (Stigler & Hiebert, 2004). The content transformed from the "four rules and regulations, domestic arithmetic, and literal arithmetic" during the pre-independence era to the immediate past of the post-independence era using the "New Mathematics for Primary Schools" in 1972, through the use of the Joint School Project (JSP) and published mathematics books by different authors and the Mathematics Association of Ghana (MAG) (Mireku, 2010). In terms of pedagogy, little attention has been paid to addressing the needs of learners with different interests in terms of pedagogical approaches. There is no differentiation in teaching mathematics to learners of varying mathematical strengths, as teacher-centred methods of rigour continue to dominate classroom teaching and learning (Mireku, 2010). These methods of rigour turned to present mathematics as dry and uninteresting to young learners and, hence, the need to make mathematics teaching and learning fun for students (Doz, 2021).

To confront this long-standing teacher dominance in the mathematics teaching-learning space, it is prescribed in the rationale of the new mathematics curriculum that mathematics teachers "must facilitate learning in the mathematics classroom" to "provide learners with the opportunities to expand, change, enhance, and modify how they view the world" (NaCCA, 2020; p.vi). This is based on the philosophy that learner-centred teaching and learning is engaging, productive, and constructive. This approach is more engaging if the teacher creates opportunities for learners to facilitate inquiry-based learning. For inquiry, learners must have the opportunity to develop their technological acumen through the use of information communication technology (ICT). In this way, students can research historical facts about mathematical concepts, thereby contextualising the learning process through knowledge construction based on the learners' skills instead of having it delivered to them dry and in rigorous forms. Mathematics is a social

construct that is out of the work of mortals, so knowing its history and using it as a pedagogical tool will motivate and help students learn mathematics better (Doz, 2021).

Understanding mathematical concepts' background and epistemological evolution undoubtedly interests and motivates students to understand such concepts rather than simply presenting them dryly. However, many students have had negative experiences with learning mathematics, so they consider it the most detestable subject. Therefore, the importance of introducing the history of mathematical concepts (HoMC) in the teaching and learning of mathematics cannot be overstated (Panasuk & Horton, 2013). Research has shown that teachers who use HoM concepts as a pedagogical tool tend to make their classrooms livelier and more engaging, and students better understand the concepts taught (Doz, 2021; Butuner & Baki, 2020). This study is, therefore, intended to explore teachers' readiness to use alternative means of HoMC to teach mathematics while calling upon curriculum planners to include it in the strategies available for teaching mathematics, as it has been well incorporated into the national framework's standards for teaching mathematics in countries such as Russia, Austria, Turkey, Brazil, Italy, France, Norway, New Zealand, and China, where it is functioning effectively (Panasuk & Horton, 2012).

2. Purpose of Study

Historical research on mathematical concepts (Cajori, 1923; Black, 1951) initially focused on understanding their development. Recently, interest shifted toward these historical insights' educational (Charalambous et al., 2009; Fauvel, 1991) and pedagogical (Doz, 2021; Azman & Maat, 2021) values. Many researchers advocate for incorporating the history of mathematical concepts (HoMC) into mathematics education, yet challenges exist, including educators lacking the expertise to blend history with teaching mathematics (Ho, 2008). Students express enthusiasm for learning through HoMC and find it enhancing understanding (Ho, 2008). Consequently, there is a growing call (Butuner, 2015) to integrate HoMC into math instruction. The effectiveness of this integration depends on how well it is used as a pedagogical tool (Vallhonesta & Massa-Esteve, 2019).

In Ghana's evolving math curriculum, raising teachers' awareness of HoMC as a pedagogical value is essential. However, research suggests this awareness is lacking (Charalambous et al., 2009). Awareness is crucial for changing educators' beliefs and practices (Palmeri, 2015). The issue of awareness was addressed with US teachers concerning their views on the history of mathematics (Bolinger-Horton & Panasuk, 2011), but awareness beyond these borders is needed. To address the struggles of mathematics teachers and find practical pedagogical approaches, we qualitatively explored the lack of awareness among Ghanaian mathematics educators regarding the integration of HoMC into mathematics instruction. The goal was to promote teachers' appreciation of HoMC's benefits and raise awareness within the mathematics teaching community about its pedagogical potential. The research result responds to a persistent challenge many math teachers face – how to engage students and enhance their understanding and interest in mathematics learning. The results of investigating the awareness and attitudes of Ghanaian educators toward using HoMC contribute valuable insights into the pedagogical landscape. By understanding the barriers that hinder educators from incorporating HoMC into their teaching practices, strategies can be developed to overcome these obstacles and provide teachers with the knowledge and tools to effectively utilise HoMC in the classroom.

Ultimately, this research was carried out to foster a greater appreciation for the history of mathematical concepts and their role in mathematics education. Educators can be empowered to make informed decisions about incorporating HoMC into their teaching practices by promoting awareness and understanding among teachers. In effect, the integration of HoMC into mathematics instruction has the potential to enrich the educational experience for both teachers and students. Based on the results of this research, the challenges and opportunities associated with this approach are explored to inspire educators to explore the rich history of mathematical concepts as a pedagogical tool in their classroom engagements.

In summary, this study aimed to explore the need to raise Ghanaian teachers' awareness of the need to integrate and use the history of mathematical concepts as a pedagogical tool in mathematics teaching. The following were the objectives of the study.

1. To find out teachers' awareness level of the HoMC and whether it is used as a pedagogical tool for teaching mathematics.
2. To find out how the HoMC, as a pedagogical tool, must be integrated into teaching mathematics.
3. To find out what level of knowledge teachers possess of the HoMC.

The research questions to accomplish the research objectives were as follows.

1. What is the awareness level of teachers of the HoMC and its use as a pedagogical tool for teaching mathematics?
2. How have teachers been integrating HoMC, as a pedagogical tool, into teaching mathematics?
3. What is the content knowledge level of teachers in the HoMC?

3. Literature Review

3.1. History of Mathematical Concepts

Every concept used in the discipline of mathematics has a historical antecedent. It evokes achievement when one is exposed to the knowledge of the history of such concepts (Clark, 2016). This knowledge dispels doubt and the abstract nature of the concepts as it humanises them, for every concept is a social construct and not a myth (Bidwell, 1993). There are many numeral systems, but only one can completely meet the demands of human society. The number zero, symbolically written as "0" in the Hindu-Arabic numeral system, resulted from human interaction, which Al-Khawarizmi formally documented, and it has now become a part of the discipline of "mathematics". The word came into English through French zéro from Italian zero, a contraction of the Venetian 'zevero', a form of Italian 'zefiro' via safira or 'sifr'. In pre-Islamic times, 'sifr' means "empty". 'Sifr' evolves to mean "zero" (Arsham, 2020). A complex number is one number of interest students will be interested in knowing how it originated because it looks abstract. It came about as a result of Al-Khawarizmi trying to solve the cubic equation, so history has it (Merino, 2006).

Al-Khawarimi was noted for the significant volumes of work done in algebra. According to history, algebra was originally Arabic, meaning "reunion of broken parts". Many students develop anxiety at the sight of algebra, but initially, Al-Khawarimi has a simple, practical approach to solving it (Syaf, 2018; Merino, 2006). Syaf reported that most students have difficulty working with algebra because they are "not careful in thinking, less persistent, less able to see the usual problems in new ways" (p.72). This anxiety will be eliminated if students are exposed to approaches such as Al-Khawarizmi's and not over-formalized procedures for working it. Hence, teachers need to know some of these histories and use them as pedagogical tools for conceptualising mathematics. His findings showed that students who participated in the activity felt satisfied and fulfilled in their work. In his work, "History of Math Concepts", Finashin (2015) enumerated a set of activities that the Pythagoras and the Medieval mathematicians have done to illustrate to us that mathematics is but a human enterprise and must not be taken "seriously" to lost sight of its aesthetic and elegance nature and the practicality of it.

Butuner (2015) demonstrated that students can effectively use historical approaches to solve mathematical problems, such as using the dissection method for volume calculations. Although some had difficulty with geometry, they found the historical method accessible, suggesting that integrating the history of mathematical concepts can improve student performance. HoM concepts, such as Zeno's paradox of Achilles and the Tortoise (Black, 1951), can also captivate students in sequences and series, showcasing the human aspect of mathematical challenges. This approach can reduce the frustration associated with mathematics problem-solving failures.

3.2. History as a Pedagogical Tool

In their review of the literature on HoMC, Butuner and Baki (2020) identified categories of reasons and their sub-divisions of why HoMC must be considered a pedagogical tool for teaching mathematics. They listed the reasons as "learning mathematics" – by viewing and identifying learners' difficulties, "views about mathematics" – that is, using HoMC as a tool to make learners more daring to confront problems, "teachers' didactic history and pedagogical accumulation" – assisting students to learn through different means available, "affective tendencies towards mathematics" – using HoMC to let learners see advantages in the new approaches used, "viewing mathematics as a cultural involvement" – using the HoMC to better the attitudes of students towards mathematics. Looking at the HoMC in this way, it allows teachers to help students learn mathematics from the sociocultural perspective, thereby not putting pressure on learners to clamp theorems and identities in their heads, leading to mental load. Azman and Maat (2021) observed that employing the HoMC in mathematics delivery also assists teachers in strengthening their content and pedagogical knowledge in the HoMC.

With all these benefits enumerated, Ho (2008) observed that several of his participants had difficulty using the HoMC effectively since they failed to transition from the historical perspective of the concept to discuss the topic itself, a situation he called the "didactical transposition", having difficulty transposing ideas in the process of discussion. Forced to teach mathematics through the HoMC in this manner, Ho (2008) observed that it might be better if they were not asked to teach it since the teachers cannot connect the historical antecedents and the discussed topic. In such a situation, it was advised that teachers were educated through professional development sessions to possess the content knowledge in HoMC and how to manoeuvre between the two – the content knowledge of the subject and the HoMC as a pedagogical tool. Fauvel (1991) agreed that teachers may become confused if confronted with using HoMC when they are not cut for it, and he professed a strategy by which teachers can conduct lesson deliveries using the HoMC as a pedagogical tool.

3.3 Integration and Use of History of Mathematical Concepts

The integration of the HoMC into teaching mathematics is the connectedness or the weaving of different cultures "into the midst of other histories with the most diverse socio-cultural qualities" (Mendes, 2020, p. 4). Here, the teacher is expected to possess the art of connecting the past to the contemporary and, even so now, on their feet so as not to meddle in the lesson being presented to the students. This is very important because once harm is caused, it is difficult to reverse. According to Mendes (2020), history plays a "decisive role in understanding the epistemological relationships established by the different kinds of mathematics with regards to their social dimensions…" (p. 5). Therefore, it is critical for the teacher to integrate the HoMC very well and use it to the advantage of learners. To this end, the participants in Burns' (2010) study agreed that the HoMC must be integrated into their teaching. The findings noted that this "positive change could be due to a better understanding of what is involved in incorporating the history of mathematics into the curriculum" p. 6. These pre-service teachers participated in planning lessons involving the history of mathematics and also researched famous mathematicians; therefore, they appreciated the importance of the HoMC as a pedagogical tool in teaching mathematics. One of the ways for a teacher to carry the class along is to inspire interest and motivation in learners to induce enthusiasm in the class. Doz (2021) suggests that teachers must integrate the HoMC into their lessons the entire length of the year to boost students' total engagement in the lesson. To successfully integrate and use HoMC, Butuner (2018) suggests the need to train teachers by giving them in-service training to raise their knowledge level in using the HoMC as a pedagogical tool.

3.4. Importance of HoMC

The importance of the HoMC cannot be ignored by anyone or any teacher who desires to create an ambience for success. According to Panasuk and Horton (2012), most teachers demonstrated a good perception of the history of mathematical concepts and believed that it had its place in the school curriculum. Since they valued the good of the HoMC, the teachers were optimistic about assisting their students in becoming more interested in it to bolster their confidence. It is worth noting that when HoMC is integrated into the mathematics classroom, teaching is more likely to attract students' interest, leading to increased motivation (Goktepe & Ozdemir, 2013). The likelihood effect is that students' interest helps them be curious and discover different approaches to solving mathematics problems. The students of Butuner and Baki (2020) came to believe that being a successful person in mathematics means being as creative as the ancient Egyptians were in calculating the volume of the pyramid. This is good for learners to increase their motivation level for higher intellectual stimulation. A person with a strong belief in a process is more motivated to perform than others, so subjects who reported having a higher belief in the HoMC were likelier to engage more in historical events and pursue problem-solving.

3.5. Teacher Knowledge in HoMC

Even though it is widely reported that the knowledge of the HoMC enriches students' understanding of mathematics (Vallhonesta & Massa-Esteve, 2019), Clark (2012) observed that not much has occurred among their subjects. Therefore, there is a need to engage deeper in research for concrete evidence. In their conclusion, however, Vallhonesta and Massa-Esteve (2019) stated that activities based on the analysis of historical facts contribute to students' knowledge of the social and scientific context of the period involved. They advised that when problems are posed, the student's cultural background must be considered. This implies that if teachers have a foretaste of knowledge in the history of mathematical concepts, they can integrate pieces of information with the awareness of their sources and the capability to discern potential inaccuracies in those sources (Palmeri, 2015) or not. If this is not done, it fails to connect relevant pieces of information to help learners.

3.6. Empirical Studies

Several studies have been done on the history of mathematical concepts and reported the benefits thereof. In Taiwan, Liu (2003) studied the need for teachers to integrate the history of mathematical concepts (HoMC) into teaching and reported that the integration develops positive attitudes and increases motivation toward mathematics. He explained that HoMC helped students appreciate and demonstrate the ability to engage in problem-solving and humanising mathematics. In Greece, Tzankis et al. (2002) found that in integrating HoMC as a pedagogical tool, one must give direct information regarding the concept and awareness in mathematics's social and cultural context. They explained that the best tools to use are primary sources, historical problems, historical packages, and history-based projects. Vallhonesta and Massa-Esteve (2019) have suggested that experiential activities and visual tools must be used to carry through with HoMC as a pedagogical tool.

Ho (2008) used a study group of 102 students from Singapore and found that the HoMC had an important effect on students: they had a better understanding and positive attitudes toward learning mathematics. Even though the students experienced these

positive attitudes, he reported that teachers were not experienced in using HoMC and, therefore, could not make relevant connections between HoMC and the topics. Teachers had challenges with the time factor and had to incorporate this into their teaching. In a related study in the US, Bolinger-Horton and Panasuk (2011) reported on 367 teachers' assessments on integrating HoMC into their teaching. It was found that those with fallibilist perspectives towards mathematics were more likely to use HoMC in their lessons than those with an absolutist view of mathematics. Thus, the fallibilists' view-holders were readier to incorporate HoMC into their lessons.

In an exposition, "History for the Teaching of Mathematics: Transformation and Mobilization of Mathematical Knowledge for School" to support the use of the HoMC, Mendes (2020) explained that "The history of mathematics is not only a history of the definition of mathematical objects but that of a creative process that involves society, culture and cognition" (p. 2). In this respect, it must be understood that the HoMC is not a set of ideas to be relayed to students but a utilisation of the critical ideas that worked in time past that work today. This didactic reinvention better suits the objectives defined by the mathematics teachers and the level of depth that the students need to learn. Therefore, he implored all mathematics teachers to explore the HoMC produced in time, past, and space and how they could reflect this in teaching mathematics today. It is, therefore, important for teachers to find stimulating ways to invite learners to investigate and understand the realities surrounding the mathematics being taught.

Using a mixed-method approach involving 94 pre-service teachers by administering questionnaires and semi-structured follow-up interviews, Charalambous et al. (2009) found that teachers needed to develop productive beliefs and attitudes toward teaching mathematics. Their results indicated that several members held more intense formalist beliefs than those they brought into the study, and their platonic beliefs became less intense at the end of the program. However, all the participants held a relatively strong belief in their ability to teach mathematics, while some held less positive attitudes toward mathematics than their initial entrance attitude. They concluded that teachers in training must be exposed to more historical knowledge of teaching mathematics since they "need help to develop the knowledge that is both useful and usable for the work of teaching". (p.170)

The methods applied by Liu (2003) and Vallhonesta and Massa-Esteve (2019) were documentary sources, whereas Tanksi et al. (2002) used content analysis; Ho (2008) used case study with qualitative approaches; Bolinger-Horton and Panasuk (2011) used independent sample t-test analysis. Although the literature showed the benefits and challenges that both students and teachers have in integrating HoMC into mathematics teaching, we demonstrated whether or not Ghanaian teachers possess the HoMC knowledge and are aware of it as an alternative pedagogical tool available for them to integrate into the teaching and learning of mathematics, thereby making mathematics lessons and their deliveries interesting, motivating and conceptually understandable to their students.

3.7. Theoretical Framework

The theoretical framework in this study is Vygotsky's social constructivism theory. This theory was used to explore the need for teachers to choose the HoMC as a pedagogical tool to influence social interactions and learners' cognitive development (Kurt, 2020). The main assertion of Vygotsky's theory is that children's cognitive development is carried through social interaction with competent adults. Thus, children are socially acculturated before they develop cognitively. So, if mathematics results from social activity (Watson, 2008), it is learned through the social perspective –its history. Since human learning is primarily a social process, interacting with students and pre-service teachers through the HoMC as a pedagogical tool (Butuner & Baki, 2020; Ho, 2008) and mathematics educators (Clark et al., 2016) is constructively productive. Social constructivism believes that knowledge and reality are constructed through discourse. The teachers need to engage and explore their learners' views and abilities to use the HoMC to teach mathematics (Saleem et al., 2021). One of the tenets of Vygotsky's theory is that language is the basis for learning, and the history of mathematical concepts is articulated through other means of language, such as the logic of reasoning, interrelatedness, and connectivity. Thus, through social interactions, the subject initially had no foundations in history since it provides intellectual rigour of proof (Chorlay et al., 2022), appealing to students, teachers, and even the general populace.

Several teachers do not know the HoMC and are unaware of its possible use as a teaching tool or are reluctant to use it because they cannot connect it with the topic being taught (Ho, 2008). However, according to Bolinger-Horton and Panasuk (2011), teachers' awareness of using the HoMC as a pedagogical tool to teach mathematics can be raised through Vygotsky's concept of scaffolding to help these teachers. Scaffolding is a temporary support given to a learner to overcome a temporary challenge, after which the learner can perform independently. On that development, Tzanakis et al. (2002) posit that HoMC can be a pedagogical tool for teaching mathematics (Haverhals & Roscoe, 2010) if teachers are aware of it and its benefits. Furthermore, teachers' awareness of using the HoMC as a pedagogical tool was shown by Karaduman (2010), who demonstrated its potency in delivering a better learner outcome for elementary students using the quantitative approach.

Thus, in Vygotsky's social constructivism theory, teachers can interact with students using the HoMC to construct meaningful ideas of the concepts from their original natural states. Through such means, teachers can connect mathematical concepts to their cultural settings of everyday life, becoming aware of their possible use as pedagogical tools for teaching these concepts.

3.8. Conceptual Framework

The framework shows the interacting variables that influence teacher awareness of using HoMC. The broken arrows show a disconnect among the variables, which is connected on purpose. A teacher can possess knowledge of HoMC but may not consider it a tool and, therefore, integrate it into teaching. However, a teacher can possess the knowledge of HoMC and consider it as a tool and, therefore, integrate it into teaching, leading to a positive effect on students' learning outcomes or students' conceptual understanding of the topic or feeling capable of constructing their knowledge.

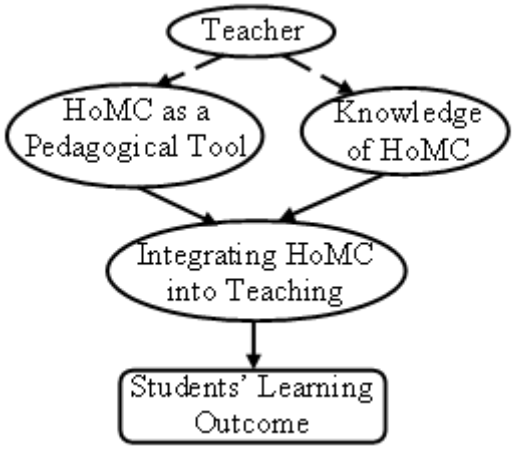

Fig. 1. Conceptual Framework of teacher awareness to use HoMC as a Tool

4. Methodology

The research paradigm employed in this study was one of interpretivism. Interpretivism is the understanding that the focus is on making "meaning of the data collected through the researcher's thinking and cognitive processing by interacting with the participants" (Kivunja & Kuyini, 2017, p. 33). Because it assumed that the phenomenon being studied was of multiple realities, these realities must be explored, and meanings must be made of them through the interactions between the participants and the researcher. The data collection instruments include interviews, discourses, text messages, reflective journals, and even body communications.

Qualitative research was employed with an inductive approach, where the study considered specific situations of individual teachers and attempted to move to the generalisation of the findings. The methodology was mono-qualitative research within a short-term period of about one month. The strategy for this study was an explorative case study of selected teachers. A survey was conducted to understand the knowledge level of participants of the history of mathematical concepts (Melnikovas, 2018). This was followed by an interview to understand how much the teachers were aware of the HoMC.

Participants were senior high secondary (SHS) school mathematics teachers at Akropong-Akuapem township in Eastern Ghana. This municipality has a large student population, hence many teachers of diverse backgrounds. There are three SHS and 37 mathematics teachers. The teachers were noticed in this study because it was the transition grade level to the tertiary grade, where students needed to be well established before moving on to the tertiary level. The SHS were the only basic schools in session when data needed to be collected.

The participants consisted of ten mathematics teachers from two schools, one with 18 and the other with 19 mathematics teachers. However, the actual number that agreed to participate in the research work was eight. All the participants had a first degree in mathematics education or its equivalent. Their years of experience ranged between 4 to 17 years. These schools were purposively selected due to their convenience regarding distance from the researcher and the time available to conduct the research. With written approval letters outlining the study's intent and procedures, permission to conduct the study was secured from the school authorities through the respective Heads of Department (HODs). Following the approval, consent for participant recruitment was also sought from the mathematics teachers through the HODs. However, recruitment faced challenges due to two factors. First, most teachers resided outside the school and only attended on working days, coinciding with their engagement periods. Second, the recruitment of teachers coincided with examination preparation and supervision duties the week before the Easter break, further limiting teacher availability. Despite these challenges, four teachers from each school volunteered to participate, totalling eight participants. After obtaining the informed consent, participants completed a semi-structured interview questionnaire and responded to ten history fact items. Immediately following the questionnaire, the interviews did not directly address the history of fact responses but rather the

history of mathematical concepts and their applicability in the teaching process. Each interview lasted between 10 and 17 minutes. The entire process took approximately one week, from initial permission to final interviews.

The data were analysed using qualitative content analysis (Elo & Kyngas, 2008). The researcher audio-recorded the interview and manually transcribed it to minimise errors. The transcribed data was then analysed using a qualitative content analysis approach. The transcribed interviews were played back to the participants to ascertain their veracity. The transcribed interviews were then color-coded for analysis.

Issues of trustworthiness were handled based on the authenticity of the following.

1. Credibility explains the limit to which a data set and its analysis are trustworthy. This was ensured by asking critical questions to exhaust all possible avenues of doubt.
2. Dependability is observing the same outcome or findings under similar conditions. To ensure this, experts scrutinised the data collection instrument to ensure that it collected the necessary data.
3. Confirmability shows how others can confirm the findings in the same study area. This was ensured by adequately representing the participants' views.
4. Transferability is where study readers can relate the findings to their context. This was ensured by ensuring that the data collected reached its saturation point where no one could doubt its authenticity and trustworthiness (Kivunja & Kuyini, 2017; Elo & Kyngas, 2008).

Further, the instruments were pilot-tested in a different school, about 5 km away from the schools of this study. Items found not desirable were eliminated or recrafted to capture the desired information.

5. Results

5.1 Awareness of Teachers of HoMC

The awareness level of teachers of the HoMC and its use as a pedagogical tool for teaching mathematics were analysed. We also investigated the significance of increasing awareness among Ghanaian schools mathematics teachers about using the history of mathematical concepts as a pedagogical tool. Specifically, we explored teachers' awareness of this history and its potential as a teaching strategy. The first research question was asked to examine whether teachers knew the history of mathematical concepts and their pedagogical use. They also assessed whether participants recognised history as a dynamic aspect of human culture and mathematics as an integral cultural component involving culturally relevant math activities (Table 1).

Table 1. Level of Awareness of Teachers of the HoMC.

Awareness level of teachers of the HoMC as a pedagogical tool for teaching mathematics			
History of mathematics as an evolving part of human culture.		Mathematics is part of our culture	Activities showing mathematics is part of our culture
Yes	Deduction of time and distances from natural elements, building styles and patterns, trading activities, and forms of every human activity, e.g., farming, drumming, etc.	Symbols, counting in our local language, designs, mode of measurements, entrepreneurship, census, constructions, etc.	Designs, buildings, walking, the process of eating, games, weather patterns, time, marketing, home economics, etc.

The participants in the study responded affirmatively to the questions and provided evidence of activities that showcased the influence of mathematics in Ghana. When asked about the relevance of the curriculum to students' real-life experiences, several participants expressed mixed opinions. They indicated that it was only sometimes or not always relevant, while others were more positive, stating that it was indeed very relevant. Those who felt that several topics were not practically applicable to students' lives mentioned difficulties relating certain concepts, such as quadratic equations, to real-life situations. For example, one participant stated, "Teaching a student quadratic equation sometimes, it is challenging to bring them to understand the real-life situations...but if you take other examples too...you can, for example, like probability, you can bring it to life situation that they will get it." Another participant noted that several concepts were part of the curriculum but did not directly or indirectly affect students' way of living, citing an example of an art student learning calculus, which they perceived as wasteful since the student was not going into engineering and would not apply it in real life. This teacher was unaware of the potential artistic benefits of learning calculus and struggled to relate it to the art student's field of study. This suggested that considering the historical aspect of the concept may not have been a priority for this teacher.

Although all the teachers knew that mathematical concepts have their historical background, they admitted not incorporating this knowledge into their teaching. The few who claimed to use historical aspects in their teaching said they did so to encourage students to study the concepts. When asked about their level of awareness regarding the history of mathematical concepts and how much of it they could use as a pedagogical strategy, the participants indicated a lack of knowledge in this area. For instance, one experienced teacher stated, "I don't know much... but I'll give myself like...60% to 70%. The one that I know, I use about...50% to 60% of it." This suggests that even teachers with years of experience were necessarily well-informed or effective in incorporating the history of mathematical concepts into their lessons. It is important to note that experience does not always equate to effective teaching practices, as it could also reflect an accumulation of experience in ineffective methods over time.

As to how they would use the HoMC as a form of strategy in the delivery of their presentations, it was clear that "before I introduce the topic. It is necessary to give them fair knowledge about how that very concept came about before the topic itself begins", and by "introducing, saying little about the history of that concept as an introduction to that particular concept". The participants who claimed to have a certain level of knowledge in the history of mathematical concepts saw the 'introduction' as the only place the HoMC could fit into their delivery. They did not know beyond this because it was not even a severe consideration in training the mathematics teacher. After all, "apart from the one being taught when we were doing the history of mathematics in the university…. apart from that, I don't think I know any…." It was evident here that the history taught at the university was not meant to build the pedagogical strength of the mathematics teacher to be. Although all the participants agreed to be aware of the HoMC, they could not use it to teach aspects of the mathematics content. They rarely used it as a response (Table 2). The history of mathematical concepts was not given any reverence in the school mathematics curriculum because in "the WASSCE, they will not ask of it" and "the exam system of today is making just giving the students what will help them to pass exams not what they will use in real life". Moreover, because of this, "the time factor too….." is limited to spend on 'teaching the history' of mathematics in a mathematics classroom. As such, they hardly use it because it lacks connection with reality.

Table 2. Level of Awareness of Teachers of the HoMC.

Awareness level of teachers of the HoMC and its use as a pedagogical tool for teaching mathematics		
How much of the HoMC can be used to teach mathematical concepts?	What knowledge of the history of mathematical concepts are you aware of?	How often do you use the HoMC you are aware of?
I don't know, I don't know much, I can't estimate the level of my awareness, about less than 40%	The history of the number system, the history of Pythagoras' theorem, that of Archimedes and the *pi*, series and sequence; I know of Mr. Venn, who brought the Venn diagram for the easy solving of the set. Pythagoras theorem, I know of the trigonometry history.	Not… (laughs). Not regular…I barely use them; I don't know much about them…that is why I don't duel much time on them. I mainly use them when it is related to the topic I am teaching…. if the history is relevant to a particular topic at that time, I use them, but I often don't use them in teaching.

5.2 Integrating HoMC as a Pedagogical Tool

A research question was asked to determine how HoMC could be an excellent strategic tool for teaching mathematics. To respond to that, the participants were asked how they viewed integrating HoMC into teaching mathematics. Participants viewed that integrating the HoMC into teaching mathematics was "the best thing to do" "since the history will give you more knowledge on the concepts you are teaching and how it came about…and eh…how to relate them to real-life activities. So, I think it would have been the best strategy". "It is very good… because sometimes… students normally see math as something abstract", so "If you use it. You use it well…it is very good because it will help you to develop the concept of that particular topic. It will help the students to understand that particular topic". "When students have a fair knowledge of the history when the concept is delivered, they see it not to be in an abstract form". "Yes, when you introduce the history behind it, then you are halfway through the learning of that concept". Table 3 presents a summary of the responses. Even though most of them held the view that the HoMC was helpful if used well, several thought it was "irrelevant to students" because "it is not examinable. It is just going to waste your time, so, sometimes, I ignore it because it is not relevant at their stage, so sometimes, I ignore it" (Table 3).

Table 3. Teacher integration of HoMC into teaching.

Integrating HoMC as a pedagogical tool into teaching mathematics		
How do you see integrating HoMC into teaching mathematics as a strategy?	How do you see the benefits of integrating HoMC into teaching mathematics?	What is your philosophy about the nature of mathematics?
Knowledge of the historical context can enhance the understanding and development of mathematical concepts and their real-life applications, making it a valuable teaching strategy.	As a form of clarification of the concepts being taught	Utilitarian views of mathematics

Participants were asked how they perceived mathematics to ascertain whether their inclination towards mathematics influenced their decision to integrate or not to integrate the HoMC into teaching mathematics. Their responses were as follows: "Something that we need to interact with our environment... it is not absolute", "It is an integral part of lifestyles..." "Mathematics is a way of life...", "Math is life... everything in life has math in it, there is nothing that you will do without math in it, in medicine, there is math, in eating there is math", "It is something which is practical". These responses suggested a utilitarian perspective of mathematics held by most participants. It was expected that teachers with such a perspective would develop a positive attitude toward the HoMC and recognise its relevance as an effective pedagogical tool in teaching mathematics. By incorporating the HoMC into their lesson delivery, teachers can make a more meaningful impact on students' understanding of mathematical concepts.

Based on the findings of the analysis above, it became evident that the participants lacked the necessary content knowledge for the history of mathematical concepts, as indicated in Table 4. This knowledge gap was attributed to the fact that the history of mathematical concepts was not taught to them during their initial teacher education in the universities, as it was not perceived as essential for their later years of teaching. Additionally, it was not included as an examinable aspect in the secondary school mathematics curriculum, leading to the perception that time spent on it would be "wasted".

Table 4. Demonstration of participants' content knowledge on HoMC.

Question Number	Correct Response	Wrong Response	No Response
1	-	-	8
2	3	-	5
3	3	2	3
4	3	-	5
5	3	-	5
6	1	3	4
7	1	-	7
8	-	1	7
9	1	-	7
10	3	-	5

Acknowledging the positive perception of the history of mathematical concepts (HoMC) as a valuable pedagogical tool for teaching mathematics is crucial. Therefore, it is imperative to consider incorporating the HoMC into teacher education development programs and schools' professional development sessions. This proactive approach will effectively enhance the pedagogical skills of mathematics teachers in schools, ultimately benefiting the quality of mathematics education delivered to students.

6. Discussion

The present study was based on recognising the need to raise awareness about incorporating HoMC in mathematics education as a means of innovative instruction. The qualitative research approach was employed to explore the participants' perspectives regarding their awareness levels and the integration of HoMC into their teaching practices. The findings revealed that the participants demonstrated a high level of awareness of mathematics as an evolving cultural phenomenon, acknowledging its pervasive presence in human activities. However, it was observed that the participants lacked the knowledge of HoMC as a tool and the pedagogical strategy to effectively utilise HoMC beyond merely making a cursory reference to it at the introduction stage of lesson delivery, as highlighted by previous research (Ho, 2008). This limitation was evident in the challenges faced by the participants in helping students connect mathematical concepts to real-life situations. As a result, the intended hands-on practicality of HoMC, as suggested by Butuner (2015), was not fully realised.

Furthermore, the results indicated that the participants' knowledge of HoMC was insufficient to integrate it into their pedagogical practices, raising concerns regarding their preparedness to utilise HoMC in instruction, as Fauvel (1991) noted. This finding aligned with previous research by Ho (2008) and Charalambous et al. (2009), which recommended professional development

sessions to enhance teachers' skills in utilising HoMC. The results also revealed similar challenges to those identified by Ho (2008), including resistance, lack of perceived relevance, limited time, and teachers' inadequacy, which hindered the integration of HoMC into the mathematics curriculum.

Despite the teachers' hesitancy in integrating HoMC into their instruction, they recognised its potential to make mathematics less abstract and more enjoyable for students. This finding resonates with the assertions of Clark (2016) and Bidwell (1993), who emphasised the positive impact of incorporating the history of mathematics in dispelling abstractness and humanising the subject. Moreover, the teachers acknowledged HoMC as a valuable instructional strategy, stating that it could lead to positive outcomes if effectively utilised. This finding aligned with the demands of Burns' (2010) participants and the perspectives of Panasuk and Horton's (2012) subjects, who recognised the significance of HoMC in the mathematics curriculum. However, unlike Panasuk and Horton's participants, the teachers in the present study expressed concerns about the non-inclusion and non-examinability of HoMC in the curriculum, which they perceived as limiting its instructional value.

It can be inferred that the positive attitudes exhibited by teachers towards incorporating the history of mathematical concepts into their pedagogical strategies indicate a potential recognition of the value of historical context in teaching mathematics. This aligns with the idea that incorporating historical perspectives into teaching mathematics can make mathematics less abstract and more enjoyable for students, as noted by Clark (2016) and Bidwell (1993). Furthermore, the observation that teachers with an educational background in mathematics were more receptive to the idea of integrating the history of mathematical concepts into their teaching suggests that professional development opportunities, as suggested by Ho (2008) and Charalambous et al. (2009), could be beneficial in enhancing teachers' skills in utilising historical context as a pedagogical tool.

The teachers' initial surprise and excitement during the interview interactions regarding the history of mathematical concepts were noteworthy. While several participants expressed uncertainty about the value of integrating HoMC in teaching mathematics, further explanations helped to clarify their understanding of its relevance. Teachers without a background in mathematics education were less likely to apply historical antecedents of mathematical concepts in their strategies for teaching mathematics. However, those with educational backgrounds showed positive attitudes towards incorporating HoMC as a pedagogical strategy. This finding was consistent with the results of Liu's (2003) study, which also reported positive attitudes among teachers who integrated the history of mathematical concepts into their mathematics teaching.

7. Conclusion

The study result showed the awareness level of mathematics teachers regarding the History of Mathematical Concepts (HoMC) and its potential as a pedagogical tool in teaching mathematics in Ghanaian secondary schools. While teachers acknowledged the historical background of mathematical concepts, there was a significant gap in incorporating this knowledge into their teaching practices. The findings indicate that the current mathematics curriculum in Ghana lacks emphasis on the pedagogical aspects of teaching, leading to a limited or lack of integration of historical perspectives into the mathematics classroom. Despite recognising the cultural relevance of mathematics, teachers struggled to connect historical contexts with real-life applications in their lessons. This gap highlights the need for a more comprehensive and pedagogically focused curriculum that equips teachers with the tools to integrate the HoMC into their teaching strategies effectively.

We uncovered a knowledge deficit among teachers concerning the history of mathematical concepts stemming from the absence of this content in their initial teacher education. This underscores the importance of incorporating the HoMC into teacher education development programs and professional development sessions. Such initiatives enhance the pedagogical skills of mathematics teachers, fostering a deeper understanding of the subject among students. While teachers acknowledged the potential benefits of integrating the HoMC into their teaching strategies, concerns about its relevance in examinations raised questions about its practical application. The utilitarian views held by teachers reflected a need for a paradigm shift, emphasising the intrinsic value of historical knowledge in understanding and applying mathematical concepts.

Addressing the awareness gap and integrating the HoMC into teacher education programs are vital in enriching the quality of mathematics education. Thus, it is needed to raise awareness and enhance teachers' pedagogical skills in utilising the history of mathematical concepts in mathematics instruction. Despite recognising the potential benefits of HoMC, challenges related to teachers' limited knowledge and skills exist in perceived non-relevance and curriculum constraints. The results emphasised the importance of providing professional development opportunities and addressing misconceptions to promote the effective integration of HoMC in mathematics education. Therefore, a proactive approach to curriculum development, teacher training, and professional development is demanded to contribute to a more holistic and effective mathematics education system.

Based on the findings that teachers lack the pedagogical skills to utilise HoMC beyond the introductory stages effectively, it is recommended that professional development session opportunities be provided for teachers to enhance their skills in utilising historical context as a pedagogical tool. This includes workshops, seminars, or training sessions that focus on incorporating HoMC

in mathematics instruction, providing teachers with practical strategies and resources to effectively integrate historical perspectives in their teaching practices. Creating collaborative learning communities among teachers to share ideas, experiences, and resources related to integrating HoMC into mathematics instruction must be beneficial. Teachers can collaborate through professional learning communities, online forums, or other networking platforms such as WhatsApp to discuss challenges, share best practices, and support each other in incorporating HoMC into their teaching practices. This builds a supportive community of educators who are passionate about integrating historical perspectives in mathematics education. The results of this study revealed that teachers expressed concerns about the non-inclusion and non-examinability of HoMC in the curriculum, which they perceived limits its instructional value. It is recommended that HoMC be advocated for in the mathematics curriculum at various educational levels through curriculum guidelines, standards, or recommendations. This highlights the significance of historical context in teaching mathematics and encourages teachers to integrate HoMC into their instruction.

In the future, it is necessary to explore the long-term impact of professional development opportunities on teachers' utilisation of HoMC in their instruction. This involves longitudinal studies or follow-up surveys to assess whether the skills and strategies acquired through professional development are sustained over time and result in meaningful changes in teachers' instructional practices. Further research is needed to explore teachers' beliefs and perceptions about the value, challenges, and benefits of incorporating HoMC in their teaching practices. Qualitative studies are essential to delve deeper into teachers' perspectives, experiences, and attitudes toward using historical context in their instruction and how these beliefs and perceptions influence their instructional practices.

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